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Practical limits on addressing twisted nematic displays

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Reflective twisted nematic liquid crystal displays have gained very wide user acceptance in consumer and professional applications. They are compact flat-panel displays with a very low power consumption, which modulate ambient light and as a result have very good legibility in a wide range of illumination conditions. There is a need for such low-power flat-panel displays with high information content as data displays for portable equipment.

A number of problems must be overcome if twisted nematic displays are to be made complex enough for these applications. The most important factor determining the number of display elements that can be addressed is the viability of matrix-addressing techniques. This paper explores the limits in the number of lines that can be addressed for a display operating in reflexion.

The basic operation of the twisted nematic display

The basic structure and operation of the twisted nematic display is shown in figure 1. A thin layer of liquid crystal material (6–15 μm thick) of positive dielectric anisotropy is confined between two glass panels that have a transparent, electrically conductive layer on their interior surfaces overlaid by a surface alignment layer (e.g. rubbed polyimide), which imposes a preferred alignment direction on the liquid crystal director. The director at opposite surfaces is constrained to lie orthogonal in the plane of the surfaces, imposing a 90° twist within the liquid crystal layer. Incident polarized light has its electric vector waveguided by the liquid crystal on passing through the display, so that it lines up with the polarization direction of the lower polarizer and is transmitted (figure 1*a*). On applying a field somewhat greater than the threshold voltage to the liquid crystal, the director reorients as shown in figure 1*b*, lining up with the field. The waveguiding of the twisted structure is therefore removed and polarized light is blocked by the lower polarizer, giving rise to the well known dark-on-light display.

The best display performance is achieved under direct drive, that is when each display element is individually driven. The complexity of direct-drive displays is limited by two main factors. One is the difficulty and expense of making connections to large numbers of picture elements, and the second is the cost of drive electronics, which would become prohibitive.

Addressing a display on a matrix format reduces the number of external connections and drivers from $n+1$ to a minimum of $2\sqrt{n}$ for a display of n picture elements. The motivation for matrix addressing is therefore clear.

Matrix addressing

I shall consider a matrix addressed display in which one panel carries a number of parallel conductor stripes, the rows, and the other carries a second set orthogonal to the first, the columns. Suitable voltages applied to the rows and the columns cause voltages to appear across the liquid crystal at the intersection of the row and column electrodes, resulting in some optical change.

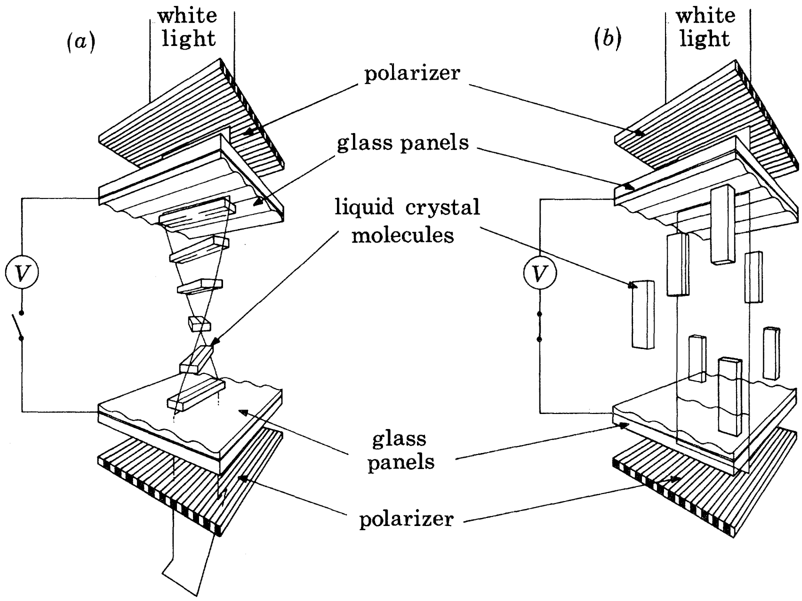

Figure 1. The twisted nematic field effect: (*a*) without field; (*b*) with field.

Static matrix addressing

The simplest way to address such a matrix is to apply various steady voltages to its row and column electrodes. The voltage across any matrix element is defined by the voltage difference between its row and column electrodes. Row and column voltages can be chosen to maximize the voltage difference between selected and unselected elements, but it is not possible to devise a scheme in which unselected elements have no voltage across them. Thus the display must have a nonlinear electro-optic response (i.e. a sharp threshold) so that unselected elements are not activated.

There are restrictions on the patterns that can be displayed by using such a static addressing scheme. For example a closed figure with a hollow centre cannot be displayed, since elements in the centre are energized at the same time as those around the perimeter. However, static addressing can sometimes be used to advantage in applications where the information displayed is restricted, such as for cross-hair graticules in optical instruments, or waveform identity addressing as devised by Shanks (1979) for oscilloscope displays.

Dynamic matrix addressing

The solution to the problem of addressing closed figures is to enter information to the display sequentially a line at a time. Figure 2 shows a schematic of such a time-multiplexed matrix addressing scheme. The information to be displayed is applied in parallel to the columns and an enable or scanning voltage (V_s) is applied to line 1. Information on the columns is then changed

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to that appropriate to line 2, and line 2 is enabled. The information is entered as the data voltage (V_d) such that $-V_d$ is applied to selected elements and $+V_d$ to unselected elements. Under these circumstances a selected element receives a pulse of magnitude $|V_s+V_d|$ while it is being addressed and pulses of magnitude $|V_d|$ while all the other lines are being addressed. Similarly an unselected element receives a pulse of magnitude $|V_s-V_d|$ while being addressed and $|V_d|$ for the remainder of the frame time. The drive voltage across any element therefore changes with time, so the transient

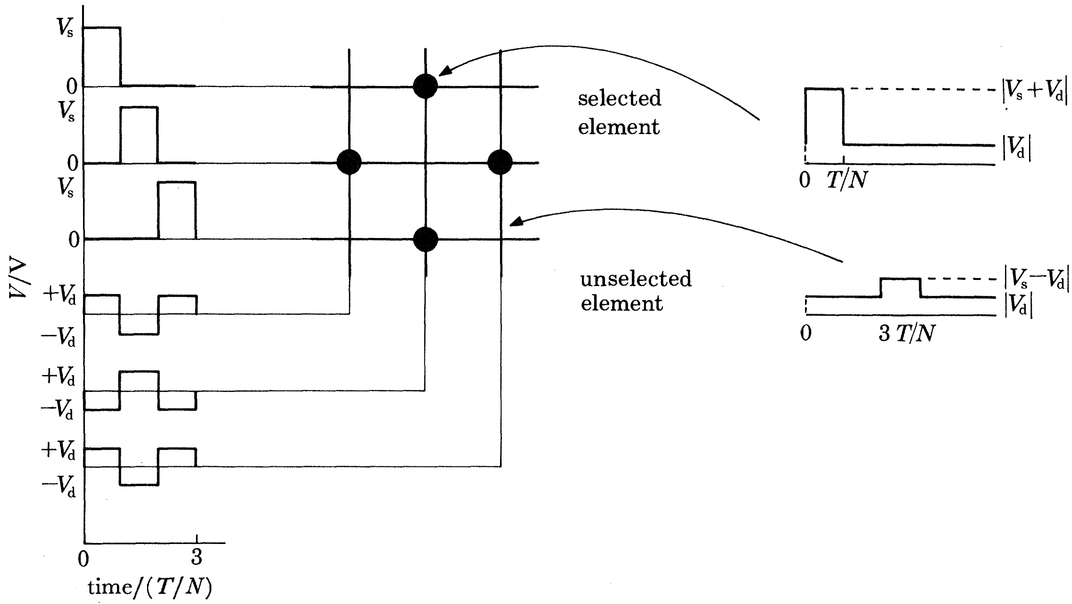

FIGURE 2. Schematic of a time-multiplexed addressing scheme (three lines in a matrix of N lines shown, frame time = T).

response characteristics of the display are important in analysing matrix performance. For the twisted nematic display, rise and fall times are similar and fairly long (tens to hundreds of milliseconds). Since the frame time for the entire display must be faster than about 25 ms to avoid perceptible flicker, the steady-state response of the display is determined by the integral over the full-frame time of the drive waveform (r.m.s. voltage). This is known as a fast-scan multiplexed addressing scheme.

The optimum voltage levels for fast-scan multiplexing

The derivation of the optimum voltage levels V_s and V_d and calculation of the resultant voltage discrimination between selected and unselected elements for a particular display matrix requires consideration of the waveform across the elements during the entire scan. This has been analysed by Alt & Pleshko (1974). By taking the general case of a matrix of N lines the r.m.s. voltages across selected (V_{ON}) and unselected elements (V_{OFF}) can be calculated:

$$V_{ON}^2 = (V_s+V_d)^2/N + V_d^2 - V_d^2/N \qquad (1)$$

and

$$V_{OFF}^2 = (V_s-V_d)^2/N + V_d^2 - V_d^2/N. \qquad (2)$$

As the number of lines N increases, V_{ON} and V_{OFF} become dominated by the data voltage V_d. This is an inevitable consequence of fast-scan addressing. Thus for any particular device the number of lines that can be scanned depends on the sharpness of the electro-optic response curve.

A schematic electro-optic response curve relating optical response to the applied voltage, known as the optical transfer characteristic (o.t.c.), is shown in figure 3. A figure of merit or voltage discrimination ratio, M, between the voltage level $\hat{V}_{ON}$ when the display is judged to be in its minimum acceptable ON condition and the voltage $\hat{V}_{OFF}$ when the display is acceptably OFF can be defined as

$$M = \hat{V}_{ON}/\hat{V}_{OFF}. \tag{3}$$

From this ratio for a given o.t.c. with a figure of merit M, the maximum number of scanned lines N_{max} and the corresponding data and scanning voltage V_d and V_s can be completely specified:

$$N_{max} = \{(M^2+1)/(M^2-1)\}^2; \tag{4}$$

$$V_d = \tfrac{1}{2}\hat{V}_{OFF}[M^2+1]^{\frac{1}{2}}; \tag{5}$$

$$V_s = \tfrac{1}{2}\hat{V}_{OFF}(M^2+1)^{\frac{3}{2}}/(M^2-1). \tag{6}$$

For a given number of scanned lines N, the optimum choice in the ratio of scanning and data voltages results in maximum r.m.s. voltage discrimination between selected and unselected elements given by

$$\hat{V}_{ON}/\hat{V}_{OFF} = \{(N^{\frac{1}{2}}+1)/(N^{\frac{1}{2}}-1)\}^{\frac{1}{2}}. \tag{7}$$

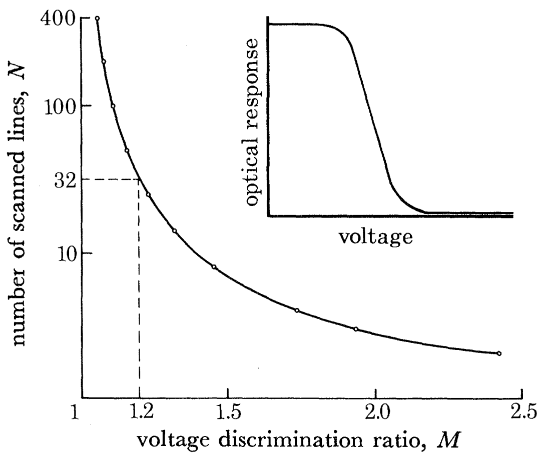

FIGURE 3. The voltage discrimination ratio, M $(= \hat{V}_{ON}/\hat{V}_{OFF})$ (defined from the optical transfer characteristic (o.t.c.)) required for an increasing number of scanned lines, N. The inset shows a schematic o.t.c.

This shows that increasing the number of scanned lines places increasingly stringent demands on the voltage discrimination ratio M of the display device (see figure 3). A large scanning capability N implies a low value for M. For successful operation with 32 lines, for example, the display must switch through its defined minimum acceptable contrast with only 20 % change in r.m.s. voltage. Thus the measurement of display contrast as a function of applied r.m.s. voltage (o.t.c.) shows the maximum number of lines N that can be addressed satisfactorily for a defined display contrast.

Factors determining the number of addressable lines for a twisted nematic display

The application of the above results to a determination of the maximum number of lines that can be addressed for a real display is not straightforward. The sharpness of the o.t.c. can be optimized for twisted nematic displays through the liquid crystal material used and through a number of aspects of display construction. In addition the voltage discrimination ratio V_{ON}/V_{OFF} may be chosen in several different ways depending on the application.

The threshold voltage of the display is viewing-angle dependent, and light travelling obliquely through the display and bisecting the two surface alignment directions will show a decreased threshold voltage compared with normal incidence along the direction of tilt of the liquid crystal molecules. Figure 4 shows the o.t.c. measured in transmitted light for normal, 10° and 45° incidence in this direction, which is known as the principal viewing plane. The viewing angle required for a particular display application, together with the minimum acceptable contrast for selected elements and the maximum acceptable contrast for unselected elements, governs the choice of V_{ON} and V_{OFF}.

The threshold voltage is also temperature-dependent, with typical shifts of 5–10 mV/°C. The temperature range specified for a particular application is therefore an added restriction in multiplexability, unless electronic temperature compensation of the threshold voltage is acceptable.

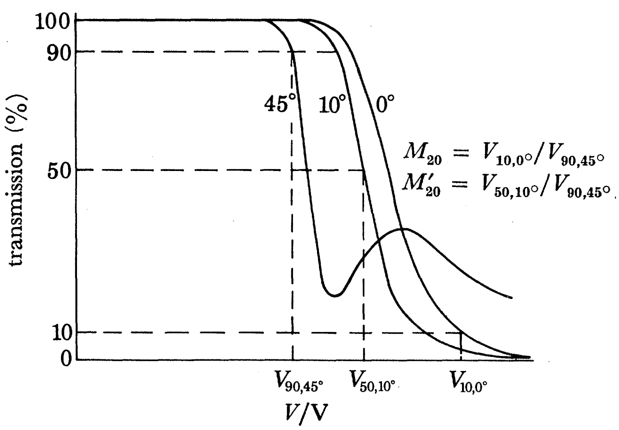

Figure 4. The optical transfer characteristic of a twisted nematic liquid crystal display measured in transmission at 0°, 10° and 45° incidence in the low threshold voltage quadrant.

The effect of display construction on multiplexability

A number of aspects of display construction influence the sharpness of the optical transfer characteristic. Their effects are widely accepted and have been summarized previously (Kahn & Birecki 1980; Birecki & Kahn 1980).

They may be summarized as a series of guidelines that should be followed to optimize contrast and viewing angle for maximum multiplexability.

(*a*) No barrier layer: a barrier layer acts as a capacitor in series with the liquid crystal layer, reducing the sharpness of the o.t.c.

(*b*) Thin low-tilt alignment layers (about 2° tilt angle): the sharpness of the threshold is reduced as the alignment tilt angle increases. However, a finite tilt angle is required to prevent reverse-tilt patches on switching.

(*c*) Minimum possible cell thickness (depends on liquid crystal birefringence – the Mauguin limit).

(*d*) Less than 90° twist angle (85°).

(*e*) No cholesteric additives: addition of a cholesteric additive alters the director configuration and optical properties of the layer.

(*f*) Alignment of the polarizer dielectric axes orthogonal to the adjacent surface alignment directions.

Optimization of liquid crystal material

A number of parameters of liquid crystal materials influence the sharpness of the o.t.c. and therefore the multiplexability. To achieve high levels of multiplexing, liquid crystals are required that have a low birefringence Δn, low splay: bend elastic constant ratio, k_{33}/k_{11}, low twist elastic constant (k_{22}) and long pitch (i.e. no cholesteric additive).

Matrix addressing limits for reflective displays

A severe limitation is imposed on the figure of merit, M, owing to shifts in the threshold voltage over the usable temperature range. Electronic temperature compensation for the threshold voltage is a requirement for achieving high levels of multiplexing. The definition of figure of merit, M, then depends only on the viewing angle and the contrast required. Conventionally this has been taken from the o.t.c.s measured in transmission.

The o.t.c. in reflexion

In practice the derivation of the o.t.c. for a reflective display is complicated by the superposition of contrast from the first and second passes through the cell. Light follows different paths through the cells on these passes, depending on the reflector and illumination conditions.

Reflective measurement system

A reflective optical bench has been built that was designed to give diffuse illumination conditions similar to those provided by diffused overhead ceiling lights, daylight, etc., in typical viewing conditions. The display cell mount is rotatable through 360° and is positioned at the focus of a hemispherical white diffusing reflector. Illumination is provided by a ring of filament lamps mounted on the base plane of the hemisphere but shielded to prevent direct illumination of the display. Front-surface reflexions of the diffusing reflector are eliminated by a black absorbing band around the viewing direction. This models the role of the head in typical viewing conditions. The display is imaged on to a detector by using a telescope and fibre-optic probe rotatable through $\pm 45°$ from normal incidence to the display.

A comparison was made between the figures of merit measured from the o.t.c.s of one display in transmission and in reflexion with a number of different reflector materials. The display cell measured was 8.5 μm thick, with a rubbed polymer surface alignment layer, and was filled with E60A (B.D.H. Chemicals Ltd). HN42 polarizers were laminated onto the display with their dichroic axes orthogonal to the adjacent surface alignment directions.

The normal incidence transfer characteristic is shifted to lower voltages when measured in reflected light, as is that measured at 10° incidence, and the oblique incidence threshold at 45° in the low-voltage quadrant is increased slightly. As a consequence the multiplexing figures of merit for 10 : 1 contrast at normal incidence, $V_{10,0°}/V_{90,45°}$, and for 2 : 1 contrast at 10° from normal incidence, $V_{50,10°}/V_{90,45°}$ (defined for less than 1.1:1 contrast for unselected elements at 45° incidence, as shown in figure 4), are lower by approximately 15 %, increasing the apparent multiplexing capability. The increase in threshold voltage at oblique angles of incidence is because some of the light reflected in this direction had its first pass in the high threshold voltage quadrant. The reduction in threshold voltage observed at 10° and normal incidence is also observed in the high-voltage quadrant and the two adjacent quadrants. This is due to the shadow cast on the reflector by a first pass through the display in the low-voltage direction. Thus diffuse reflected illumination has the effect of considerably widening the viewing angle of a multiplexed display in addition to causing an apparent increase in threshold sharpness. Both specular and fairly diffuse reflectors show very similar results.

Berman & Chan (1980) investigated the role of the shadow cast on the reflector on the viewability of twisted nematic displays under direct drive, and came to similar conclusions.

These factors are important in assessing the ultimate limits of matrix addressing of practical reflective twisted nematic displays.

Assessment of a multiplexed twisted nematic display

A 14-way multiplexed display was chosen for this purpose. This is a 160-character display of 2.8 mm character height (four lines of 40 characters on a 7 × 5 dot matrix) developed at S.T.L. Two rows of characters (14 lines) are addressed from the top of the display and two rows from the bottom. It was designed for use in home or office environments for a viewing angle between 0° and 45° with electronic temperature compensation.

Aspects of cell construction were optimized to take full advantage of developments in multiplexable liquid crystal materials. A polyimide alignment layer was chosen because its conformal properties give advantages in forming a barrier in very thin layers (25–40 nm). This was rubbed unidirectionally to give a low tilt angle (2°) alignment in directions biased to give an 85° twist angle in the assembled display. The display was sealed to produce a cell spacing of about 10 μm and filled with a selected liquid crystal material. Polarizers were laminated onto the front and rear surfaces with their dichroic axes aligned orthogonal to the adjacent surface alignment directions.

The choice of liquid crystal material was dictated by the overriding requirement for a sharp threshold. It has been found (Bradshaw & Raynes 1982) that the addition of alkyl esters to biphenyls decreases k_{33}/k_{11}, particularly for long-chain alkyl esters (Sturgeon, this symposium). The main limitation for such materials is the temperature of formation of smectic phases. The dielectric anisotropy is also decreased, leading to a higher threshold voltage. This may lead to an increased threshold sharpness, but generally the effects are expected to be small (Kahn & Birecki 1980). The higher threshold and resultant operating voltage were acceptable for our application.

Two liquid crystal multiplexing materials that are mixtures of long-chain alkyl esters and biphenyls were developed in cooperation with B.D.H. Chemicals. The data for these materials for 11 μm thick cells are shown in table 1. The first of these materials, LC14, was developed for the 14-way multiplexed display, which requires a voltage discrimination ratio ($V_{ON}/V_{OFF} = M$) of 1.315. As discussed previously, the choice of V_{ON} and V_{OFF} depends on the requirements of the user. The specified viewing angle for the display was 0° to 45°. Two figures of merit are taken from the optical transfer characteristics measured in transmission. M_{20} (20 °C) is defined for 10:1 contrast at normal incidence for selected elements and less than 1.1:1 contrast for unselected elements at 45°. M'_{20} is defined for a reduced contrast of 2:1 at 10° from normal incidence for selected elements. M_{20} was 1.68 and M'_{20} was 1.31. In reflexion these are reduced to about 1.39 and 1.16 respectively, owing to shadowing effects.

A number of these reflective displays were assessed under 14-way multiplexed drive in diffuse ambient illumination provided by overhead fluorescent lamps and daylight. Almost full display contrast was achieved from normal incidence out into the principal viewing plane for selected elements (i.e. $V_{ON} \approx V_{10,0°}$). Unselected elements were judged to have unacceptably high contrast for angles greater than 45° in the principal viewing plane (i.e. $V_{OFF} \approx V_{90,45°}$), but in spite of crosstalk the display was readable out to 55° incidence. Owing to the influence of the shadow the display was easily readable with better than 2:1 contrast out to glancing incidence in all directions outside the principal viewing plane.

Table 1. Measurements on two liquid crystal mixtures in cells 11 μm thick at 20 °C

	LC14	LC21
smectic–nematic transition temperature/°C	−20	−10
nematic–isotropic transition temperature/°C	60	60
birefringence (Δn)	*ca.* 0.146	*ca.* 0.146
V_{10}, normal incidence	3.46	3.34
V_{50}, 10° incidence	2.70	2.69
V_{90}, 45° incidence	2.06	2.11
$M_{20} = V_{10}/V_{90}$	1.68	1.58
$M'_{20} = V_{50}/V_{90}$	1.31	1.27
$-\mathrm{d}V_{90}/\mathrm{d}t$ (0–40 °C)/(mV/°C)	20	25

Relating this performance to the measured figures of merit in reflexion for the liquid crystal materials used, we would expect M_{20} to describe the threshold sharpness. M_{20} is 1.39, implying a maximum number of multiplexable lines, N_{max}, of 10. Experience with viewer trials shows that this underestimates N_{max} for a practical reflective display, illustrating the difficulty of judging perceived display characteristics on the basis of simple physical measurements. The alternative figure of merit, M'_{20}, measured in transmission (1.31 for LC14) predicts 14-way multiplexing for a 2:1 contrast from 10° incidence out into the principal viewing plane; this is clearly an underestimate of contrast and viewing angle for a practical reflective display. In practice M'_{20} is a convenient figure of merit to use because the underestimation of N_{max} due to transmissive measurements and the overestimation due to lower contrast and viewing angle compensate, so that it predicts N_{max} for good contrast between 0° and 45° for a reflective display.

The variation of threshold voltage with temperature for LC14 is larger than for most commercially available liquid crystals, but is fairly linear. As a result, electronic temperature compensation with a thermistor was found to be adequate.

LC21 was a mixture developed later with a larger proportion of the long-chain alkyl esters. This has an improved threshold sharpness at the expense of a shorter nematic temperature range and a larger variation of threshold voltage with temperature. An M'_{20} of 1.27 implies a maximum number of scanned lines of 18; 21-way multiplexing for a 6-line alphanumeric display would require an M'_{20} of 1.25. However, it is judged that with LC21 a 21-way multiplexed reflective display can be made with good contrast over a slightly narrower viewing angle than our 14-way multiplexed display and with contrast greater than 2:1 to glancing incidence outside the principal viewing plane. Because of the larger nonlinear change of threshold voltage with temperature, the thermistor sensor may be inadequate for electronic temperature compensation, and more sophisticated methods such as those proposed by Hilsum *et al.* (1978) may be necessary.

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Limits in matrix addressing of twisted nematic displays

Kahn & Birecki (1980) derived performance limits for optimized fast-scan multiplexed twisted nematic displays by means of computer calculations of the optical transfer characteristics. They investigated the effect on their figures of merit of altering one parameter at a time, neglecting the effects of temperature, and derived any improvement in multiplexing that could be achieved from optimizing them. Their baseline was a 3° tilt angle, 90° twisted cell 8 μm thick, filled with E7 and doped with cholesteric to prevent reverse twist. The liquid crystal parameters varied were k_{22}, k_{33}/k_{11}, $\Delta\epsilon$ and Δn. In addition they varied cell thickness and twist angle, and investigated the effect of cholesteric dopants and polarizer orientation.

They estimated the upper limit for multiplexing for a theoretical material with all parameters optimized by adding improvements for each onto the baseline value for multiplexability. For the range of parameters investigated this requires $\Delta n = 0.1$, $k_{33}/k_{11} = 0.2$, $k_{22} = 2 \times 10^{-12}$ N and a pitch of infinity (no cholesteric dopant). The upper limit was $N = 24$ for temperature-compensated direct-view displays with more than 2:1 contrast between normal incidence and 40° in the principal viewing plane. These conditions were determined on the basis of measurements in transmitted light, but they state that viewing angles should be better in reflexion.

Table 2. Material parameters and multiplexability of a number of liquid crystal mixtures

	LC14	LC21	E120	E130	E140
T_{SN}/°C	−20	−10	< −20	< −20	< −20
T_{NI}/°C	60	60	60	64.3	63
Δn (589.6 nm, 20 °C)	0.146	0.146	0.146	0.154	0.147
k_{33}/k_{11}†	*ca.* 0.95	*ca.* 0.9	1.04	0.88	0.90
M'_{20}	1.31‡	1.27‡	1.29§	1.27§	1.22§
N_{max}	14	18	16	18	25

† Reduced temperature of 0.9.
‡ Measured at 11 μm (S.T.L.).
§ Measured at 7 μm (B.D.H.).

We have found that display contrast, viewing angle and apparent threshold sharpness are considerably improved in reflexion. It is therefore useful to relate the results obtained for practical displays in reflexion to these calculated limits of performance. A number of material parameters of high-level multiplexing liquid crystals, including LC14, LC21 and more recent developments from B.D.H. (E120–E140), are listed in table 2 with their multiplexing figures of merit. The elastic constant ratios k_{33}/k_{11} of E120–E140 were kindly provided by M. J. Bradshaw at R.S.R.E. The effect of reducing k_{33}/k_{11} and the birefringence Δn on increasing the threshold sharpness is apparent. Relating these parameters to Kahn & Birecki's results and substituting for k_{33}/k_{11} and Δn, a maximum of approximately 10 lines would be expected for LC14 from their figure of merit for a contrast greater than 2:1 between 0° and 45°. In practice the 14-way multiplexed reflective display containing LC14 has high contrast within this angle and more than 2:1 contrast to glancing incidence.

Assuming that liquid crystal materials may be developed in the future with parameters approaching the Kahn & Birecki optimum, up to 14 more lines may be multiplexed than can be achieved with materials such as LC14. Displays multiplexed 32 ways have been demonstrated with acceptable viewing characteristics for many applications. The viewing cone of the displays

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is reduced to about 30°. On this basis 46 lines or more may ultimately be addressable, and 32 lines are addressable with existing materials.

Materials such as LC21 and E140 (B.D.H.), which have been developed more recently, have figures of merit M'_{20} that imply that 18 lines can be multiplexed in cells 10–11 μm thick and 25 lines in cells 7 μm thick with the same contrast and viewing angles as the 14-way multiplexed display described. These show that progress is being made towards ultimately achieving 46 lines or more of multiplexing for twisted nematic displays.

Conclusions

It has been shown in this paper that reflective-mode twisted nematic displays operating with good contrast over a 30° viewing cone may ultimately be multiplexed over about 46 lines. This is based on the assumption that liquid crystal materials will be developed with optimum characteristics ($k_{33}/k_{11} = 0.2$, $k_{22} = 2 \times 10^{12}$ N, $\Delta n = 0.1$) for multiplexing. The number is derived from the results of Kahn & Birecki (1980), taking account of the difference in characteristics of displays measured in transmission and reflexion and related to viewer trials on a 14-way multiplexed display made at S.T.L. For a 7×5 dot matrix format alphanumeric display, at least 12 lines of information could be displayed by dividing the matrix and addressing half the display from each side. For applications where a more restricted viewing angle is acceptable, this may be increased further. Assuming up to 400 columns for a display the maximum information content expected from fast-scan multiplexed twisted nematic displays is likely to be of the order of 5×10^4 picture elements.

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